Week Two Problems

Epsilon Summer Series

July 4, 2014

Try to solve as many as you can. We will discuss these problems at our next meeting.

1. Given that x and y are nonzero real numbers such that $x + \frac{1}{y} = 10$ and $y + \frac{1}{x} = \frac{5}{12}$, find all possible values of x.

2. Let O_1 and O_2 be concentric circles with radii 4 and 6, respectively. A chord AB is drawn in O_1 with length 2. Extend AB to intersect O_2 in points C and D. Find CD.

3. There are 100 students who want to sign up for the class Introduction to Acting. There are three class sections for Introduction to Acting, each of which will fit exactly 20 students. The 100 students, including Alex and Zhu, are put in a lottery, and 60 of them are randomly selected to fill up the classes. What is the probability that Alex and Zhu end up getting into the same section for the class?

4. Find the integer closest to
$$\frac{1}{\sqrt[4]{5^4+1} - \sqrt[4]{5^4-1}}$$
.

5. Point P and line ℓ are such that the distance from P to ℓ is 12. Given that T is a point on ℓ such that $PT = 13$, find the radius of the circle passing through P and tangent to ℓ at T.

6. There are 10 people who want to choose a committee of 5 people among them. They do this by first electing a set of 1, 2, 3, or 4 committee leaders, who then choose among the remaining people to complete the 5-person committee. In how many ways can the committee be formed, assuming that people are distinguishable (Two committees that have the same members but different sets of leaders are considered to be distinct.)

7. Let
$$A = \frac{1}{6}((\log_2(3))^3 - (\log_2(6))^3 - (\log_2(12))^3 + (\log_2(24))^3)$$
Compute 2^A.

8. ABC is a triangle such that $BC = 10$, $CA = 12$. Let M be the midpoint of side AC. Given that BM is parallel to the external bisector of $\angle A$, find the area of triangle ABC. (Lines AB and AC form two angles, one of which is $\angle BAC$. The *external bisector* of $\angle A$ is the line that bisects the other angle.)

9. Bob writes a random string of 5 letters, where each letter is either A, B, C, or D. The letter in each position is independently chosen, and each of the letters A, B, C, D is chosen with equal probability. Given that there are at least two A's in the string, find the probability that there are at least three A's in the string.