Three Dimensional Geometry

## Epsilon Summer Series

July 3, 2015

1 Introductory Problems

1. (a) How many triangular faces does a pyramid with 10 edges have?
   (b) How many triangular faces does a pyramid with 1000 edges have? What about n edges?

2. A cube is inscribed in a sphere. Find the ratio of the surface area of the sphere to the surface area of the cube.

3. Draw (or describe) the following shapes: Tetrahedron, cube, prism, octahedron, icosahedron, dodecahedron

4. The sum of the lengths of all the edges of a cube is 144 inches. What is the number of inches in the length of a diagonal of the cube?

5. A regular tetrahedron has a height of 20 inches. What is the length, in inches, of a lateral edge of the tetrahedron?

6. A cone-shaped mountain has its base on the ocean floor and has a height of 8000 feet. The top 1/8 of the mountain is above water. What is the depth of the ocean at the base of the mountain, in feet?

2 Bigger Challenges

1. Centers of adjacent faces of a unit cube are joined to form a regular octahedron. What is the volume of this octahedron?

2. Bhairav Volcano, shaped like a hexagonal pyramid, is 9000 feet tall. However, after a major eruption the top 2000 feet are blown off. What fraction of the volume of Bhairav Volcano still remains?

3. Triangle PAB and square $ABCD$ are in perpendicular planes. Given that $PA = 3, PB = 4$ and $AB = 5$, what is PD?
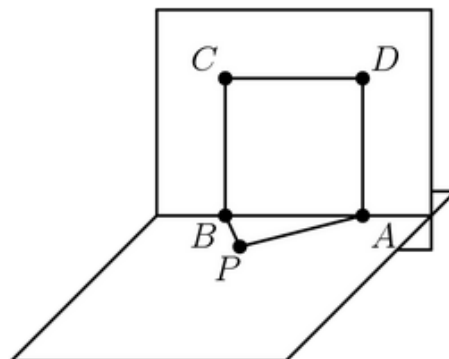
4. A pyramid with a square base is cut by a plane that is parallel to its base and is 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

5. Given a regular tetrahedron $ABCD$ with center O, find $\sin \angle AOB$.

6. There are two spheres and a cube oriented in space such that every vertex of the cube is on the larger sphere and each face of the cube is tangent to the smaller sphere. Let R be the radius of the larger sphere and let r be the radius of the smaller sphere. If the volume of the cube is 64, find the value of $R^2 + 2r$.

7. A cylindrical barrel with radius 4 feet and height 10 feet is full of water. A solid cube with side length 8 feet is set into the barrel so that the diagonal of the cube is vertical. The volume of water thus displaced is v cubic feet. Find v^2

8. A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into the two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a + b + c$?