

Expected Value

Epsilon Summer Series

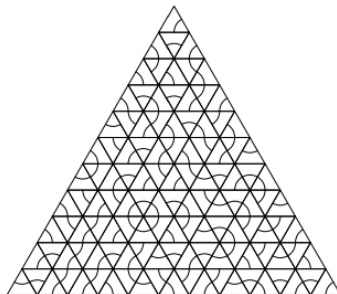
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1 Introductory Problems

- (a) At the San Diego County Fair, Eshaan is running a casino. A certain game requires you to flip a coin, and pays out \$1000 if you flip heads and \$100 if you flip tails. The game costs \$600 to play. What is the expected value of your winnings? Is it worth it?
(b) Eshaan realized that no one was playing his game, so he decided to lower the price of playing the game to only \$200. However, he is now sneakily using a biased coin which turns heads 10% of the time and tails 90% of the time. What is the expected value of your winnings under this new setup? Should you play?
- (a) I roll one green die and two blue die. What is the expected value of the sum of the values on the blue dice minus the value of the green die?
(b) I do the same with 99 green die and 100 blue die. What is the expected value now?
- A bag contains 99 black beans and 1 white bean. Beans are drawn one at a time from the bag, without replacement, until the white bean is drawn. Find the expected number of black beans drawn out of the bag before the white bean is drawn.

2 Bigger Challenges

- A 10-digit binary number with four 1's is chosen at random. What is its expected value?
- Suppose that 7 boys and 13 girls line up in a row. Let S be the number of places in the row where a boy and a girl are standing next to each other. For example, for the row $GBBGGGBGBGGGBGBGGGBG$ we have $S = 12$. Find the expected value of S . (AHSME 1989).
- An equilateral triangle of side length of 11 is tiled with 121 congruent equilateral triangles such there are 11 smaller triangles along each side of the large triangle, as shown below. For each of the small equilateral triangles, we randomly choose a vertex V of the triangle and draw an arc with that vertex as center connecting the midpoints of the two sides of the small triangle with V as an endpoint. Find the expected value of the number of full circles formed. (USAMTS)



- If five squares of a 3×3 board initially colored white are chosen at random and blackened, what is the expected number of edges between two squares of the same color? (Berkeley Math Tournament 2013).
- Two jokers are added to a 52 card deck and the entire stack of 54 cards is shuffled randomly. What is the expected number of cards that will be between the two jokers? (HMMT)

3 Answers

3.1 Introductory Problems

1. (a) \$550 (b) \$190
2. (a) 3.5 (b) 3.5
3. $\frac{99}{2}$

3.2 Bigger Challenges

1. $\frac{2047}{3}$
2. $\frac{91}{10}$
3. $\frac{5}{81}$
4. $\frac{16}{3}$
5. $\frac{52}{3}$