Diagnostic Test

Epsilon Summer Series

June 25, 2015

1. Compute the smallest positive integer that is 3 more than a multiple of 5 and twice a multiple of 6.

2. A four-digit number is formed by repeating a two-digit number: for example, 2525 or 6868. What is the largest integer that divides all such integers?

3. We simultaneously flip a penny, a nickel, and a dime. What is the expected value of the sum of the values of the coins hat land on heads?

4. The sides of a triangle have lengths 15, 20, and 25. Let us say that the shortest altitude is K times the longest altitude. What is the value of K?

5. In $\triangle ADE$, $\angle ADE = 140°$, points B and C lie on sides AD and AE, respectively, and points A, B, C, D, E are distinct. If lengths AB, BC, CD, and DE are all equal, then what is the measure of $\angle EAD$, in degrees?

6. You come across the following card:

> On this card exactly one statement is false.
>
> On this card exactly two statements are false.
>
> On this card exactly three statements are false.
>
> On this card exactly four statements are false.

How many false statements are there?

7. You face a vocabulary test - there are 100 questions and a word bank of 100 words. You guess randomly, using each word exactly once. What is the expected value of the number of questions you get right?

8. Let x and y be real numbers such that $x + y = 5$ and $x^3 + y^3 = 71$. Determine $\frac{1}{x} + \frac{1}{y}$.

9. The only prime factors of an integer n are 2 and 3. If the sum of the divisors of n (including 1 and itself) is 1240, find n.

10. John is hitchhiking from San Diego to San Francisco. The probability he will see a car within the next 20 minutes is 609/625. What is the probability that he will see a car within the next 5 minutes? Assume that the probability of seeing a car at any moment is uniform for the entire 20 minutes.

11. The number 2^{29} is a 9-digit number with distinct digits. Which digit is missing?

12. In triangle ABC the medians AD and CE have lengths 18 and 27, respectively, and $AB = 24$. Extend CE to intersect the circumcirlce of ABC at F. What is the area of triangle AFB?

1 Answers

1. 48

2. 101

3. 8

4. $\frac{3}{5}$

5. 10

6. 3

7. 1

8. $\frac{25}{18}$

9. 432

10. $\frac{3}{5}$

11. 4

12. $8\sqrt{55}$