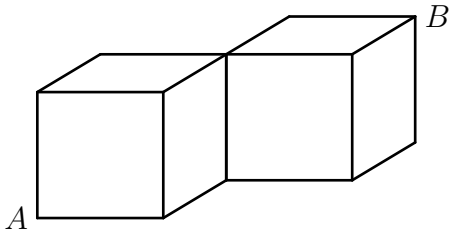
Contest 5

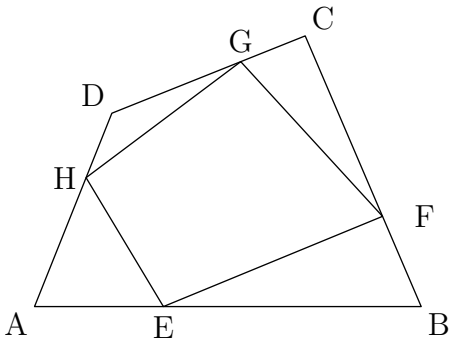
Epsilon Summer Series

July 23, 2015

1. If $m > 0$ and the points $(m, 3)$ and $(1, m)$ lie on a line with slope m, compute m.
2. The ratio of w to x is $4 : 3$, of y to z is $3 : 2$ and of z to x is $1 : 6$. What is the ratio of w to y?
3. The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is $75°$, then what is the largest angle?
4. Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?
5. Let f be the function defined by $f(x) = ax^2 - \sqrt{2}$ for some positive a. If $f(f(\sqrt{2})) = -\sqrt{2}$ then what is the value of a?
6. Compute $\sqrt{1 + 8 + 27 + \ldots + 216}$.
7. Let the roots of $x^3 - x + 1 = 0$ be a, b, c. Find the value of
$$a(1 - a) + b(1 - b) + c(1 - c)$$
8. Find the greatest integer $n < 1000$ such that $n^2 - 1$ is a power of 2.
9. Two unit cubes share an edge such that all the edges are parallel to the xyz-axes. An ant must travel from vertex A to vertex B, as shown, but it is limited to travelling on the surfaces of the cubes. Determine its shortest path.



10. In convex quadrilateral $ABCD$, angle $\angle BAD = 60°$ and sides $AB = 39, BC = 27, CD = 23$, and $DA = 22$. Points E, F, G, H are chosen on sides $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ such that $\frac{AE}{EB} = \frac{BF}{FC} = \frac{CG}{GD} = \frac{DH}{HA} = k$ and the area of $EFGH$ is exactly 52% of the area of $ABCD$. If $k > 1$, compute k.



1 Answers

1. $\sqrt{3}$
2. 16/3
3. 105
4. 12
5. $\sqrt{2}/2$
6. 21
7. -2
8. 3
9. $\sqrt{13}$
10. 3/2