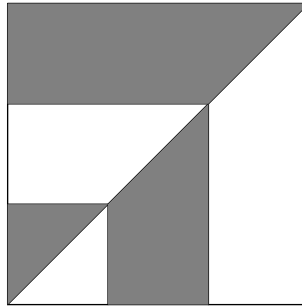# Contest 4

Epsilon Summer Series

July 16, 2015

1. Compute the fraction of the large square that is shaded.

2. Compute the value of

$$\frac{2^2}{2^{-2}} - 2^{2^2} + 2$$

3. Phil the woodchuck is chopping logs. If it takes him 15 minutes to cut a log into 6 pieces, how long in minutes does it take him to cut it into 12?

4. A triangle with integer sides has perimeter 8. What is its area?

5. Little Red Riding Hood is trying to get from the woods, located at the origin, to her grandmother's house, located at (4,2). She can only travel up and right and must stay on the lattice grid. However, she cannot pass through the point (2,1) because the wolf lives there. How many paths are there from the woods to her grandmother's house?

6. How many numbers from 100 to 999 have 3 digits that are either strictly increasing or strictly decreasing? (For example, 237 would have strictly increasing digits, 872 would have strictly decreasing digits, and 223 would have neither).

7. How many different patterns can be made by shading exactly two of the nine squares in a 3×3 grid? Patterns that can be matched by flips and/or turns are not considered different. For example, the patterns shown below are not considered different.
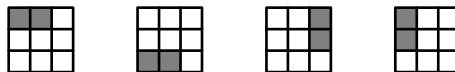
8. Define the function g so $g(a) = (a+1)^2 - a^2$. Compute

$$g(1) + g(2) + ... + g(12)$$

9. A function f from the integers to the integers is defined as follows:

$$f(n) = \begin{cases} n + 3 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

Suppose k is odd and $f(f(f(k))) = 27$. What is the sum of the digits of k?

10. Find a if a and b are integers such that $x^2 - x - 1$ is a factor of $ax^{17} + bx^{16} + 1$.

1 Answers

1. $\frac{1}{2}$
2. 2
3. 33
4. $2\sqrt{2}$
5. 6
6. 204
7. 8
8. 168
9. 105
10. 987

1. $\frac{1}{2}$
2. 2