Contest 3

Epsilon Summer Series

July 9, 2015

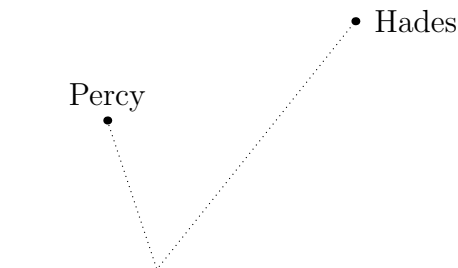1. For integers a, b and c, define $\boxed{a, b, c}$ to mean $a^b - b^c + c^a$. Compute $\boxed{1, -1, 2}$.

2. A positive number x has the property that $x\%$ of x is 4. What is x?

3. Compute
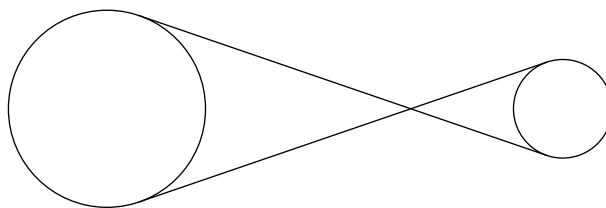$$\binom{10}{0}\binom{5}{5} + \binom{10}{1}\binom{5}{4} + \cdots + \binom{10}{5}\binom{5}{0}$$

4. Compute the area of the triangle bounded by the lines $y = x, y = -x$ and $y = 6$.

5. Percy Jackson is standing at $(0, 5)$ in the Cartesian plane; the shoreline of the River Styx is given by the line $y = 0$. He needs to run to the shoreline, become invincible, and then defeat Hades, who is lurking at (5,7). What is the length of the shortest possible path he can take?

6. What is the decimal value of last digit of 5^{38} in base 63?

7. A belt is wound around two circular drums of radius 8 and 4, whose centers are situated 37 units apart. What is the sum of the lengths of the straight-line portions of the belt?

8. Let a, b, c be the roots of $x^3 - 3x^2 + 17x - 2015 = 0$. Compute the value of $a(a+b) + b(b+c) + c(c+a)$.

9. In a class of 10 distinguishable students, 3 distinct superlatives (Funniest, Best Looking, and Most Likely to be President) are awarded at the end of the year. How many ways can this happen, given that Eshaan, Tristan, Zilu, and Bhairav, who are all members of the class, receive at least one superlative between them? The same person can receive more than one superlative.

10. Let S be the set of integers between 1 and 2^{40} whose binary expansions have exactly two 1's. If a number is chosen at random from S, find the probability that it is divisible by 9.

1 Answers

1. 2
2. 20
3. 3003
4. 36
5. 13
6. 25
7. 70
8. -8
9. 784
10. 133/780