

Contest 2

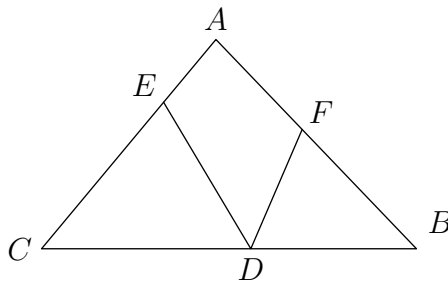
Epsilon Summer Series

July 3, 2015

1. There are 16 coins in a bank. If the coins are all nickels and dimes and they total \$1.05, how many nickels are there?
2. Find the sum

$$\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \dots$$

3. How many odd positive integers are factors of 480?
4. In triangle ABC , $AB = AC$ and $\angle A = 80^\circ$. If points D , E , and F lie on sides BC , AC and AB , respectively, and $CE = CD$ and $BF = BD$, then what does $\angle EDF$ equal?



5. If $a + b = 1$ and $a^2 + b^2 = 2$, find $a^4 + b^4$.
6. Let $P(x)$ and $Q(x)$ be polynomials of degree 3 or less such that the sum of the coefficients of P is 6 and the sum of the coefficients of Q is 7. If the leading coefficient of P is $\frac{4}{7}$, find the sum of the coefficients of the polynomial $P(x)Q(x)$.
7. Let $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ be an angle such that $\cos(\sin \theta) = 1$. Compute $\sec(\tan \theta)$.
8. Bob has a bag containing 5 blue marbles and 3 red marbles. He draws randomly without replacement until only marbles of a single color remain in the bag, at which point he stops. What is the probability that the last marble he draws is blue?
9. In rectangle $ABCD$, side $AB = 12$ and side $BC = 25$. Point E lies on side BC such that $BE < CE$ and angle $\angle AED$ is right. Compute the area of triangle CDE .
10. How many real numbers x satisfy the equation $\frac{1}{5} \log_2 x = \sin(5\pi x)$?

1 Answers

1. 11
2. $\frac{3}{16}$
3. 4
4. 50°
5. $\frac{7}{2}$
6. 42
7. 1
8. $\frac{3}{8}$
9. 96
10. 159